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$$\beta_M = \frac{\gamma r_0}{2\mu_B}\sigma \cdot \mathbf{M}_\perp = D_M \sigma \cdot \mathbf{M}_\perp$$

```
dm_eq: $$
\beta_M = \frac{\gamma r_o}{2\mu_B} \sigma \cdot \mathbf{M}_\perp
   = D_\mathbf{M} \sigma \cdot M_\perp
$$
```

$$I(q) = background + P(q)S(q)$$

```
fract_core_eq1: $$
I(q) = \text{background} + P(q)S(q)
$$
```

$$S(q) = \frac{D_f \Gamma(D_f - 1) sin((D_f - 1) tan^{-1}(q\xi))}{(qr_c)^{D_f}(1 + \frac{1}{q^2\xi^2})^{\frac{D_f - 1}{2}}}$$

```
frac_core_eq3:$$
S(q) = \frac{D_f\Gamma(D_f - 1) \sin((D_f -1) \tan^{-1}(q\xi))}
   {(qr_c)^{D_f} (1+\frac{1}{q^2\xi^2})^{(D_f-1)/2}}
$$
```

$$P(q) = \frac{P_0(q)}{V} = \frac{1}{V} F(q) F^*(q)$$

```
image001:$$
P(q) = \frac{P_o(q)}{V} = \frac{1}{V} F(q) F^*(q)
$$
```

$$F(q) = \iiint dV \rho(r) e^{-iq \cdot r}$$

```
image002:$$
F(q) = \iiint dV \rho(r) e^{-iqr}
$$
```

$$I(q) = \Phi P(q)$$

```
image003: $$
I(q) = \Phi P(q)
$$
```

$$
I(q) = \frac{scale}{V} \cdot \left[\frac{3V(\Delta\rho)(\sin(qr) - qr\cos(qr))}{(qr)^3}\right]^2 + bkg
$$

```
image004: $$
I(q) = \frac{\text{scale}}{V} \cdot \left[ \frac{3V(\Delta\rho)(\sin(qr) - qr\cos(qr))}
   {(qr)^3}\right]^2 + \text{background}
$$
```

$$
I(q) = (1-x) f_1^2(q) S_{11}(q) + 2[x(1-x)]^{1/2} f_1(q) f_2(q) S_{12}(q) + x f_2^2(q) S_{22}(q)
$$

```
image006:$$
I(q) = (1-x)f_1^2(q) S_{11}(q) + 2[x(1-x)]^{1/2} f_1(q)f_2(q)S_{12}(q) + x\,f_2^2(q)S_{22}(q)
$$
```

$$
\begin{aligned}
x &= \frac{(\phi_2 + \phi)\alpha^3}{(1-(\phi_2/\phi) + (\phi_2/\phi)\alpha^3)}, \\
\phi &= \phi_1 + \phi_2 = total\_volume\_fraction. \\
\alpha &= R_1/R_2 = size\_ratio
\end{aligned}
$$

```
image007:$$
\begin{eqnarray}
   x &=& \frac{(\phi_2 + \phi)\alpha^3}{(1-(\phi_2/\phi) + (\phi_2/\phi)\alpha^3)^2} \\
   \phi &=& \phi_1 + \phi_2 = \text{total volume fraction} \\
   \alpha &=& R_1/R_2 = \text{size ratio}
\end{eqnarray}
$$
```

$$
q = \sqrt{q_x^2 + q_y^2}
$$

```
image008:$$
q = \sqrt{q_x^2 + q_y^2}
$$
```

$$
I(q) = \frac{scale}{V} (\Delta\rho)^2 A^2(q) S(q) + bkg
$$

```
image010:$$
I(q) = \frac{\text{scale}}{V} (\Delta\rho)^2 A^2(q) S(q) + \text{background}
$$
```

$$A(q) = \frac{3[\sin(qR) - qR\cos(qr)]}{(qR)^3}\exp\left(\frac{-(\sigma_{fuzzy} q)^2}{2}\right)$$

image011:$$
A(q) = \frac{3[\sin(qR) - qR\cos(qr)]}{(qR)^3} \exp\left(\frac{-(O_\text{fuzzy} q)^2}{2}\right)
$$

$$P(q) = \frac{scale}{V_s}\left[3V_c(\rho_c - \rho_s)\frac{[\sin(qr_c) - qr_c\cos(qr_c)]}{(qr_c)^3} + 3V_s(\rho_s - \rho_{solv})\frac{[\sin(qr_s) - qr\cos(qr_s)]}{(qr_s)^3}\right]^2 + bkg$$

image013:$$
P(q) = \frac{\text{scale}}{V_s}\left[
  3V_c(\rho_c-\rho_s) \frac{[\sin(qr_c)-qr_c\cos(qr_c)]}{(qr_c)^3} +
  3V_s(\rho_s-\rho_{\text{solvent}}) \frac{[\sin(qr_s) - qr\cos(qr_s)]}{(qr_s)^3}\right]^2
  + \text{background}
$$

$$P(q) = \frac{scale}{V_{shell}}\left[\frac{3V_1(\rho_1 - \rho_2)J_1(qR_1)}{qR_1} + \frac{3V_2(\rho_2 - \rho_{solv})J_1(qR_2)}{qR_2}\right]^2 + bkg$$

image017:$$
P(q) = \frac{\text{scale}}{V_\text{shell}} \left[
   \frac{3V_1(\rho_1 - \rho_2)J_1(qR_1)}{qR_1}
  + \frac{3V_2(\rho_2-\rho_{\text{solvent}}) J_1(qR_2)}{qR_2}\right]^2
  + \text{background}
$$

$$P(q) = [f]^2/V_{particle}$$

image022:$$
P(q) = [f]^2/V_\text{particle}
$$

$$f = f_{core} + \sum_{shelli=1}^{N} f_{shelli} + f_{solvent}$$

image023:$$
f = f_\text{core} + \sum_{\text{shell}_i}^N f_{\text{shell}_i} + f_\text{solvent}
$$

$$f = 4\pi\int_0^\infty \rho(r)\frac{\sin(qr)}{qr}r^2\,dr$$

image024:$$
f = 4 \pi \int_0^\infty \rho(r) \frac{sin(qr)}{qr} r^2 dr
$$

$$
\begin{aligned}
f_{core} &= 4\pi \int_0^{r_{core}} \rho_{core} \frac{\sin(qr)}{qr} r^2 dr \\
&= 3\beta_{core} V(r_{core}) \left[\frac{\sin(qr_{core}) - qr_{core}\cos(qr_{core})}{(qr_{core})^3} \right]
\end{aligned}
$$

image025:$$
\begin{eqnarray}
f_\text{core} &=& 4\pi \int_0^{r_\text{core}} \rho_\text{core} \frac{\sin(qr)}{qr}r^2 dr \\
&=& 3\beta_\text{core} V(r_\text{core}) \left[
\frac{\sin(qr_\text{core})-qr_\text{core}\cos(qr_\text{core})}{(qr_\text{core})^3} \right]
\end{eqnarray}
$$

$$
f_{shelli} = 4\pi \int_{r_{shelli-1}}^{r_{shelli}} \rho_{shelli}(r) \frac{\sin(qr)}{qr} r^2 dr
$$

image026:$$
f_{\text{shell}_i} = 4\pi \int_{r_{\text{shell}_{ i-1}}}^{r_{\text{shell}_i}}
\rho_{\text{shell}_i}(r)\frac{\sin(qr)}{qr}r^2dr
$$

$$
\begin{aligned}
f_{solvent} &= 4\pi \int_{r_N}^{\infty} \rho_{solvent} \frac{\sin(qr)}{qr} r^2 dr \\
&= -3\rho_{solvent} V(r_N) \left[\frac{\sin(qr_N) - qr_N\cos(qr_N)}{(qr_N)^3} \right]
\end{aligned}
$$

image027:$$
\begin{eqnarray}
f_\text{solvent} &=& 4\pi \int_{r_N}^\infty \rho_\text{solvent}\frac{\sin(qr)}{qr}r^2 dr \\
&=& -3\rho_\text{solvent}V(r_N)\left[\frac{\sin(qr_N) - qr_N\cos(qr_N)}{(qr_N)^3} \right]
\end{eqnarray}
$$

$$
\rho_{shelli}(r) \begin{cases} = B\exp\left(\dfrac{A(r - r_{shelli-1})}{\Delta t_{shelli}} \right) + C & for\ \ A \neq 0 \\ = \rho_{in} = constant & for\ \ A = 0 \end{cases}
$$

image028:$$
\rho_{\text{shell}_i}(r) = \begin{cases}
B \exp\left(\frac{A(r-r_{\text{shell}_{i-1}})}{\Delta t_{\text{shell}_i}}\right) + C,
& \text{if}\ A \ne 0\\
\rho_{in} = \text{constant}, & \text{if}\ A = 0
\end{cases}
$$

$$
\begin{aligned}
f_{shelli} &= 4\pi \int_{r_{shelli-1}}^{r_{shelli}} \left[Bexp\left(\frac{A(r - r_{shelli-1})}{\Delta t_{shelli}}\right) + C \right] \frac{\sin(qr)}{qr} r^2\, dr \\
&= 3BV(r_{shelli}) \left[e^A \left\{ \frac{\alpha_{out}\sin(\beta_{out}) - \beta_{out}\cos(\beta_{out})}{(\alpha_{out}{}^2 + \beta_{out}{}^2)\beta_{out}} - \frac{(\alpha_{out}{}^2 - \beta_{out}{}^2)\sin(\beta_{out}) - 2\alpha_{out}\beta_{out}\cos(\beta_{out})}{(\alpha_{out}{}^2 + \beta_{out}{}^2)^2\beta_{out}} \right\} \right] \\
&\quad -3BV(r_{shelli-1}) \left[\left\{ \frac{\alpha_{in}\sin(\beta_{in}) - \beta_{in}\cos(\beta_{in})}{(\alpha_{in}{}^2 + \beta_{in}{}^2)\beta_{in}} - \frac{(\alpha_{in}{}^2 - \beta_{in}{}^2)\sin(\beta_{in}) - 2\alpha_{in}\beta_{in}\cos(\beta_{in})}{(\alpha_{in}{}^2 + \beta_{in}{}^2)^2\beta_{in}} \right\} \right] \\
&\quad +3CV(r_{shelli}) \left[\frac{\sin(\beta_{out}) - \beta_{out}\cos(\beta_{out})}{\beta_{out}{}^3} \right] \\
&\quad -3CV(r_{shelli-1}) \left[\frac{\sin(\beta_{in}) - \beta_{in}\cos(\beta_{in})}{\beta_{in}{}^3} \right]
\end{aligned}
$$

image029:

$$
f_{shelli} \approx 3(\rho_{out} - \rho_{in})V(r_{shelli}) \left[\frac{r_{shelli}{}^2\beta_{out}{}^2\sin(\beta_{out}) - (\beta_{out}{}^2 - 2)\cos(\beta_{out})}{\beta_{out}{}^4} \right]
$$

image030:

$$
-3(\rho_{out} - \rho_{in})V(r_{shelli-1}) \left[\frac{r_{shelli-1}{}^2\sin(\beta_{in}) - (\beta_{in}{}^2 - 2)\cos(\beta_{in})}{\beta_{in}{}^3} \right]
$$

image031:

$$
+3\rho_{out}V(r_{shelli}) \left[\frac{\sin(\beta_{out}) - \beta_{out}\cos(\beta_{out})}{\beta_{out}{}^4} \right]
$$

image032:

$$-3\rho_{in}V(r_{shelli-1})\left[\frac{\sin(\beta_{in})-\beta_{in}\cos(\beta_{in})}{\beta_{in}{}^{3}}\right]$$

image033:

$$f_{shelli}=3\rho_{in}V(r_{shelli})\left[\frac{\sin(\beta_{out})-\beta_{out}\cos(\beta_{out})}{\beta_{out}{}^{3}}\right]$$

image034:

$$-3\rho_{in}V(r_{shelli-1})\left[\frac{\sin(\beta_{in})-\beta_{in}\cos(\beta_{in})}{\beta_{in}{}^{3}}\right]$$

image035:

$$B=\frac{\rho_{out}-\rho_{in}}{e^{A}-1},\qquad C=\rho_{in}-B,\qquad V(a)=\frac{4\pi}{3}a^{3}$$
$$\alpha_{in}=A\frac{r_{shelli-1}}{\Delta t_{shelli}},\quad \alpha_{out}=A\frac{r_{shelli}}{\Delta t_{shelli}}$$
$$\beta_{in}=qr_{shelli-1},\qquad and\qquad \beta_{out}=qr_{shelli}$$

image036:

$$P(q)=[f]^{2}/V_{particle}$$

image037:

$$V_{particle}=V(r_{shellN}).$$

image038:

$$q=\sqrt{q_x{}^{2}+q_y{}^{2}}$$

image040:

$$f=f_{core}+\sum_{inter\_i=0}^{N}f_{inter_i}+\sum_{flat\_i=1}^{N}f_{flat\_i}+f_{solvent}$$

image043:

$$\begin{aligned}f_{core}&=4\pi\int_0^{r_{core}}\rho_{core}\frac{\sin(qr)}{qr}r^{2}\,dr\\&=3\rho_{core}V(r_{core})\left[\frac{\sin(qr_{core})-qr_{core}\cos(qr_{core})}{(qr_{core})^{3}}\right]\end{aligned}$$

image044:

$$f_{inter\_i} = 4\pi \int_{\Delta t_{flat\_i-1}} \rho_{inter\_i}(r) \frac{\sin(qr)}{qr} r^2 dr$$

image045:

$$f_{shelli} = 4\pi \int_{\Delta t_{inter_i}} \rho_{flat_i} \frac{\sin(qr)}{qr} r^2 dr$$

image046:

$$= 3\rho_{flat_i} V(r_{inter_i} + \Delta t_{inter_i}) \left[\frac{\sin\left(q\left(r_{inter_i} + \Delta t_{inter_i}\right)\right) - q\left(r_{inter_i} + \Delta t_{inter_i}\right)\cos\left(q\left(r_{inter_i} + \Delta t_{inter_i}\right)\right)}{\left(q\left(r_{inter_i} + \Delta t_{inter_i}\right)\right)^3} \right]$$

image047:

$$-3\rho_{flat_i} V(r_{inter_i}) \left[\frac{\sin(qr_{inter_i}) - qr_{inter_i}\cos(qr_{inter_i})}{(qr_{inter_i})^3} \right]$$

image048:

$$\rho_{inter\_i}(r) \begin{cases} = B\exp\left(\frac{\pm A\left(r - r_{flat\_i}\right)}{\Delta t_{inter\_i}}\right) + C & for\ A \neq 0 \\ = B\left(\frac{\left(r - r_{flat\_i}\right)}{\Delta t_{inter\_i}}\right) + C & for\ A = 0 \end{cases}$$

image049:

$$\rho_{inter\_i}(r) \begin{cases} = \pm B\left(\frac{r - r_{flat\_i}}{\Delta t_{inter\_i}}\right)^A + C & for\ A \neq 0 \\ = \rho_{flat_i+1} & for\ A = 0 \end{cases}$$

image050:

$$\rho_{inter\_i}(r) \begin{cases} = B\ \mathrm{erf}\left(\frac{A\left(r - r_{flat_i}\right)}{\sqrt{2}\ \Delta t_{inter\_i}}\right) + C & for\ A \neq 0 \\ = B\left(\frac{\left(r - r_{flat\_i}\right)}{\Delta t_{inter\_i}}\right) + C & for\ A = 0 \end{cases}$$

image051:

$$f_{inter_i} = 4\pi \int_{\Delta t_{flat_i - 1}} \rho_{inter_i}(r) \frac{\sin(qr)}{qr} r^2 dr$$

image052:

$$= 4\pi \sum_{j=0}^{npts_{inter}-1} \int_{r_j}^{r_{j+1}} \rho_{inter_i}(r_j) \frac{\sin(qr)}{qr} r^2 dr$$

image053:

$$\approx 4\pi \sum_{j=0}^{npts_{inter}-1} \left[\begin{array}{r} 3\left(\rho_{inter_i}(r_{j+1}) - \rho_{inter_i}(r_j)\right) V\left(r_{sub_{layer_j}}\right) \left[\frac{r_j{}^2 \beta_{out}{}^2 \sin(\beta_{out}) - (\beta_{out}{}^2 - 2)\cos(\beta_{out})}{\beta_{out}{}^4}\right] \\ -3\left(\rho_{inter_i}(r_{j+1}) - \rho_{inter_i}(r_j)\right) V\left(r_{sub_{layer_{j-1}}}\right) \left[\frac{r_{j-1}{}^2 \sin(\beta_{in}) - (\beta_{in}{}^2 - 2)\cos(\beta_{in})}{\beta_{in}{}^3}\right] \\ +3\rho_{inter_i}(r_{j+1}) V(r_j) \left[\frac{\sin(\beta_{out}) - \beta_{out}\cos(\beta_{out})}{\beta_{out}{}^4}\right] \\ -3\rho_{inter_i}(r_j) V(r_j) \left[\frac{\sin(\beta_{in}) - \beta_{in}\cos(\beta_{in})}{\beta_{in}{}^3}\right] \end{array} \right]$$

image054:

$$V(a) = \frac{4\pi}{3} a^3$$

$$\alpha_{in} \sim \frac{r_j}{(r_{j+1} - r_j)}, \quad \alpha_{out} \sim \frac{r_{j+1}}{(r_{j+1} - r_j)}$$

$$\beta_{in} = qr_j, \quad and \quad \beta_{out} = qr_{j+1}$$

image055:

$$P(q, \alpha) = \frac{scale}{V} f^2(\boldsymbol{q}) + bkg$$

image 059: in cylinder.py

$$f(\boldsymbol{q}) = 2(\Delta\rho)V \sin(qL\cos\alpha / 2) / (qL\cos\alpha / 2) \frac{J_1(qr\sin\alpha)}{(qr\sin\alpha)}$$

image060: in cylinder.py

$$P(q) = \frac{scale}{V} \int_0^{\pi/2} f^2(q, \alpha) \sin\alpha \, d\alpha + bkg$$

image063: in cylinder?

$$P(q) = \int_0^{2\pi} d\varphi \int_0^{\pi} p(\theta,\varphi) P_0(q,\alpha) \sin\theta \, d\theta$$

image064: in cylinder.py (but different limits of integration -- check!)

$$P(q,\alpha) = \frac{scale}{V_s} f^2(\mathbf{q}) + bkg$$

image067: in cylinder.py

$$f(\mathbf{q}) = 2(\rho_c - \rho_s) V_c \sin[qL\cos\alpha/2] / [qL\cos\alpha/2] \frac{J_1[qr\sin\alpha]}{[qr\sin\alpha]}$$

$$+2(\rho_s - \rho_{solv}) V_s \sin[q(L+2t)\cos\alpha/2] / [q(L+2t)\cos\alpha/2] \frac{J_1[q(r+t)\sin\alpha]}{[q(r+t)\sin\alpha]}$$

image068:

$$P(q) = (scale) V_{shell} (\Delta\rho)^2 \int_0^1 \Psi^2[q, R_{shell}(1-x^2)^{1/2}, R_{core}(1-x^2)^{1/2}] \left[\frac{\sin(qHx)}{qHx}\right]^2 dx$$

$$\Psi(q,y,z) = \frac{1}{1-\gamma^2}[\Lambda(qy) - \gamma^2 \Lambda(qy)]$$

$$\Lambda(a) = 2J_1(a)/a$$

$$\gamma = R_{core}/R_{shell}$$

$$V_{shell} = \pi(R_{shell}{}^2 - R_{core}{}^2)L$$

image072:

$$I(q) = N \int_0^{\pi/2} [\Delta\rho_t (V_t f_t(q) - V_c f_c(q)) + \Delta\rho_c V_c f_c(q)]^2 S(q) \sin\alpha d\alpha + background$$

image081:

$$\Delta\rho_i = \rho_i - \rho_{solvent}$$

image082:

$$\langle f_t^2(q) \rangle_\alpha = \int_0^{\pi/2} \left[\left(\frac{\sin(q(d+h)\cos\alpha)}{q(d+h)\cos\alpha} \right) \left(\frac{2J_1(qR\sin\alpha)}{qR\sin\alpha} \right) \right]^2 \sin\alpha d\alpha$$

$$\langle f_c^2(q) \rangle_\alpha = \int_0^{\pi/2} \left[\left(\frac{\sin(qh\cos\alpha)}{qh\cos\alpha} \right) \left(\frac{2J_1(qR\sin\alpha)}{qR\sin\alpha} \right) \right]^2 \sin\alpha d\alpha$$

image083:

$$S(q) = 1 + \frac{2}{n}\sum_{k=1}^{n}(n-k)\cos(kDq\cos\alpha)\exp\left[-k(q\cos\alpha)^2\sigma_D/2\right]$$

image084:

$$P(q) = \frac{scale}{V_p}\int_0^1 \phi_Q\left(\mu\sqrt{1-\sigma^2}, a\right)\left[S(\mu c\sigma/2)\right]^2 d\sigma$$

$$\phi_Q(\mu, a) = \int_0^1 \left\{S\left[\mu/2\cos(\frac{\pi}{2}u)\right]\cdot S\left[\mu a/2\sin(\frac{\pi}{2}u)\right]\right\}^2 du$$

$$where \quad S(x) = \frac{\sin x}{x}, \mu = qB$$

image088:

$$\Delta\rho_i = \rho_i - \rho_{solvent}$$

image089:

$$V = ABC + 2t_A BC + 2t_B AB$$

image095:

$$I(q) = \frac{scale}{V_{cyl}}\int d\psi\int d\phi\int p(\theta,\phi,\psi)F^2(\mathbf{q},\alpha,\psi)\sin\theta d\theta + bkg$$

image099:

$$F(q,\alpha,\psi) = 2\frac{J_1(a)}{a}\cdot\frac{\sin(b)}{b}$$

$$a = q\cdot\sin(\alpha)[r_{major}^2\sin^2(\psi) + r_{\min or}{}^2\cos(\psi)]^{1/2}$$

$$b = q\frac{L}{2}\cos(\alpha)$$

image100:

$$I(q) = \frac{scale}{V}(\Delta\rho)^2 < A^2(q) > +background$$

image106: in capped_cylinder.py

$$
A(Q) = \pi r^2 L \frac{\sin[(QL/2)\cos\theta]}{(QL/2)\cos\theta} \frac{2J(Qr\sin\theta)}{Qr\sin\theta}
+ 4\pi R^3 \int_{-h/R}^{1} dt \cos[Q\cos\theta(Rt + h + L/2)]
\times (1-t^2) \frac{J_1[QR\sin\theta(1-t^2)^{1/2}]}{QR\sin\theta(1-t^2)^{1/2}}
$$

image107: in capped_cylinder.py

$$
V = \pi r_c^2 L + 2\left[\pi\left(\frac{2R^3}{3} + R^2 h - \frac{h^3}{3}\right)\right]
$$

image108: note: equivalent is in capped_cylinder.py (different formula, same value)
$$
V = \pi r_c^2 L + 2\pi\left(\tfrac23R^3 + R^2h-\tfrac13h^3\right)
$$

$$
R_g^2 = \left[\tfrac{12}{5}R^5 + R^4\left(6h + \tfrac{3}{2}L\right) + R^3\left(4h^2 + L^2 + 4Lh\right)
+ R^2\left(3Lh^2 + \tfrac{3}{2}L^2 h\right) + \tfrac{2}{5}h^5 - \tfrac{1}{2}Lh^4 - \tfrac{1}{2}L^2h^3
+ \tfrac{1}{4}L^3 r^2 + \tfrac{3}{2}Lr^4\right]\left(4R^3 + 6R^2h - 2h^3 + 3r^2L\right)^{-1}
$$

image109: in capped_cylinder.py

$$
I(q) = \frac{scale}{V}(\Delta\rho)^2\left\langle A^2(q)\right\rangle + bkg
$$

image113: in capped_cylinder.py

$$
A(\mathbf{Q}) = \pi r^2 L \frac{\sin[(QL/2)\cos\theta]}{(QL/2)\cos\theta} \frac{2J_1(Qr\sin\theta)}{Qr\sin\theta}
+ 4\pi R^3 \int_{-h/R}^{1} \mathrm{d}t \cos[Q\cos\theta(Rt + h + L/2)]
\times (1-t^2) \frac{J_1[QR\sin\theta(1-t^2)^{1/2}]}{QR\sin\theta(1-t^2)^{1/2}},
$$

image114: in capped_cylinder.py

$$
V = \pi r_c^2 L + 2\left[\frac{\pi}{3}(R-h)^2(2R+h)\right]
$$

image115: in capped_cylinder.py

$$R_g^2 = \left[\tfrac{12}{5}R^5 + R^4\left(6h + \tfrac{3}{2}L\right) + R^3\left(4h^2 + L^2 + 4Lh\right) + R^2\left(3Lh^2 + \tfrac{3}{2}L^2h\right) + \tfrac{2}{5}h^5 - \tfrac{1}{2}Lh^4 - \tfrac{1}{2}L^2h^3 + \tfrac{1}{4}L^3r^2 + \tfrac{3}{2}Lr^4\right]\left(4R^3 + 6R^2h - 2h^3 + 3r^2L\right)^{-1}$$

image116: in capped_cylinder.py

$$f(q) = \frac{3(\Delta\rho)V(\sin[qr(R_a,R_b,\alpha)] - qr\cos[qr(R_a,R_b,\alpha)]}{[qr(R_a,R_b,\alpha)]^3}$$

image119: in ellipsoid?

$$r(R_a,R_b,\alpha) = \left[R_b^{\,2}\sin^2\alpha + R_a^{\,2}\cos^2\alpha\right]^{1/2}$$

image120: in ellipsoid?

$$P(q) = \frac{scale}{V}\int_0^1 \left|F(q,r_{\min},r_{\max},\alpha)\right|^2 d\alpha + background$$

$$\left|F(q,r_{\min},r_{\max},\alpha)\right| = V\Delta\rho\cdot(3j_1(u)/u)$$

$$u = q\left[r_{maj}^{\,2}\alpha^2 + r_{\min}^{\,2}(1-\alpha^2)\right]^{1/2}$$

$$where\quad j_1(u) = (\sin x - x\cos x)/x^2$$

image126: in ellipsoid?

$$P(q) = \frac{scale}{V_{ell}}\int_0^1\int_0^1 \phi^2\left\{q\left[a^2\cos^2(\pi x/2) + b\sin^2(\pi x/2)(1-y^2) + c^2y^2\right]\right\}dxdy$$

$$where\quad \phi^2(x) = 9\left(\frac{\sin x - x\cos x}{x^3}\right)^2$$

image128: in triaxial ellipse?

$$P(q) = \frac{scale}{V_{ell}}\int_0^1\int_0^1 \phi^2\left\{q\left[a^2\cos^2(\pi x/2) + b\sin^2(\pi x/2)(1-y^2) + c^2y^2\right]\right\}dxdy$$

$$where\quad \phi^2(x) = 9\left(\frac{\sin x - x\cos x}{x^3}\right)^2$$

image129: in triaxial ellipse?

$$I(q) = 2\pi \frac{P(q)}{\delta q^2}$$

image133:

$$P(q) = \frac{2\Delta\rho^2}{q^2}(1 - \cos(q\delta))$$

image134:

$$P(q) = \frac{4}{q^2}\{\Delta\rho_H[\sin[q(\delta_H + \delta_T)] - \sin(q\delta_T)] + \Delta\rho_T \sin(q\delta_T)\}^2$$

image136:

$$I(q) = 2\pi \frac{P(q)}{2(\delta_H + \delta_T)q^2}$$

image136: (again --- different image type?)

$$P(q) = \frac{4}{q^2}\{\Delta\rho_H[\sin[q(\delta_H + \delta_T)] - \sin(q\delta_T)] + \Delta\rho_T \sin(q\delta_T)\}^2$$

image137: dup of image136a

$$I(q) = 2\pi \frac{P(q)S(q)}{\delta q^2}$$

image139:

$$S(q) = 1 + 2\sum_{1}^{N-1}\left(1 - \frac{n}{N}\right)\cos(qdn)\exp\left(-\frac{2q^2d^2\alpha(n)}{2}\right)$$

image140:

$$\alpha(n) = \frac{\eta_{cp}}{4\pi^2}(\ln(\pi n) + \gamma_E)$$

$$\gamma_E = 0.5772156649 = Euler's \quad const.$$

$$\eta_{cp} = \frac{q_o^2 k_B T}{8\pi\sqrt{KB}} = Caille \quad const.$$

image141:

$$P(q) = \frac{4}{q^2}(\Delta\rho_H[\sin[q(\delta_H + \delta_T)] - \sin(q\delta_T)] + \Delta\rho_T \sin(q\delta_T))^2$$

image143:

$$I(q) = 2\pi(\Delta\rho)^2\Gamma_m \frac{P_{bil}(q)}{q^2} Z_N(q)$$

image145:

$$P_{bil}(q) = \left(\frac{\sin(qt/2)}{qt/2}\right)^2$$

image146:

$$N_L = x_N N + (1 - x_N)(N+1)$$

image147:

$$I(q) = \frac{scale}{V_p} V_{lattice} P(q) Z(q)$$

image149:

$$V_{lattice} = \frac{4\pi}{3}\frac{R^3}{D^3}$$

image150:

$$\Delta a = gD$$

image151:

$$\frac{qD}{2\pi} = \sqrt{h^2 + k^2 + l^2}$$

image153:

$$\begin{array}{cccccc} q/q_0 & 1 & \sqrt{2} & \sqrt{3} & \sqrt{4} & \sqrt{5} \\ Indices & (100) & (110) & (111) & (200) & (210) \end{array}$$

image154:

$$I(q) = \frac{scale}{V_p} V_{lattice} P(q) Z(q)$$

image158:

$$V_{lattice} = \frac{16\pi}{3} \frac{R^3}{\left(D\sqrt{2}\right)^3}$$

image159:

$$\Delta a = gD$$

image160:

$$\frac{qD}{2\pi} = \sqrt{h^2 + k^2 + l^2}$$

image162:

$$\begin{array}{llllll} q/q_0 & 1 & \sqrt{4/3} & \sqrt{8/3} & \sqrt{11/3} & \sqrt{4} \\ Indices & (111) & (200) & (220) & (311) & (222) \end{array}$$

image163:

$$I(q) = \frac{scale}{V_p} V_{lattice} P(q) Z(q)$$

image167:

$$\begin{array}{llllll} q/q_0 & 1 & \sqrt{2} & \sqrt{3} & \sqrt{4} & \sqrt{5} \\ Indices & (110) & (200) & (211) & (220) & (310) \end{array}$$

image169:

$$\mathrm{I(Q)} = \frac{\mathrm{A}}{\mathrm{Q^n}} + \frac{\mathrm{C}}{1 + (|\mathrm{Q} - \mathrm{Q_0}| \xi)^m} + \mathrm{B}$$

image172:

$$\begin{aligned} I(q) &= scale \times D(x) + bck \\ D(x) &= 2\left(e^{-x} + x - 1\right) / x^2 \\ x &= (qR_g)^2 \end{aligned}$$

image172: (different extension?)

$$\mathrm{I(Q)} = \frac{\mathrm{A}}{\mathrm{Q^n}} + \frac{\mathrm{C}}{1 + (|\mathrm{Q} - \mathrm{Q_0}| \xi)^m} + \mathrm{B}$$

image174:

$$I(Q) = \frac{A}{Q^n} + \frac{C}{1 + (Q\xi)^m} + B$$

image176:

$$I(q) = scale/\left(1 + (qL)^2\right) + bck$$

image178:

$$I(q) = \frac{scale \cdot L^3}{(1 + (qL)^2)} + bck$$

$$scale = 8\pi\phi(1 - \phi)(\Delta\rho)^2$$

image180:

$$I(q) = scale \times |q|^{-m} + bck$$

image182:

$$I(q) = \frac{1}{a + c_1 q^2 + c_2 q^4} + bck$$

image184:

$$I(q) = P(q)S(q) + bck$$

$$P(q) = scale \times V(\rho_{block} - \rho_{solvent})^2 F(qR_0)^2$$

$$F(x) = \frac{3[\sin(x) - x\cos(x)]}{x^3}$$

$$V = \frac{4}{3}\pi R_0^3$$

$$S(q) = 1 + \frac{D_f \Gamma(D_f - 1)}{\left[1 + 1/(q\xi)^2\right]^{(D_f - 1)/2}} \frac{\sin\left[(D_f - 1)\tan^{-1}(q\xi)\right]}{(qR_0)^{D_f}}$$

image186:

$$I(q) = I_G(0)\exp(-q^2 \Xi^2/2) + I_L(0)/(1 + q^2\xi^2)$$

image189:

$$I(Q) = \frac{G}{Q^s}\exp\left[\frac{-Q^2 R_g^{\ 2}}{3-s}\right] \ for \ \ Q \le Q_1$$

$$I(Q) = \frac{\mathrm{D}}{Q^m} \ \ for \ \ Q \ge Q_1.$$

image191:

$$I(q) = K\frac{q^2 + k^2}{4\pi L\alpha^2}\frac{1}{1 + r_0^{\ 2}(q^2 + k^2)(q^2 - 12hC_a / b^2)} + background$$

$$k^2 = 4\pi L(2C_s + \alpha C_a)$$

$$r_0^{\ 2} = \frac{1}{\alpha\sqrt{C_a}(b / \sqrt{48\pi L_b})}$$

image191: (again)

$$Q_1 = \frac{1}{R_g}\sqrt{\frac{(m-s)(3-s)}{2}}$$

image192:

$$I(q) \ = \ I_0 \ \exp(-R_g^{\ 2} q^2 \ / \ 3)$$

image192: (again)

$$I(Q) = \frac{G}{Q^s}\exp\left[\frac{-Q^2 R_g^{\ 2}}{3-s}\right] \ for \ \ Q \le Q_1$$

$$I(Q) = \frac{\mathrm{D}}{Q^m} \ \ for \ \ Q \ge Q_1.$$

image193:

$$Q_1 = \frac{1}{R_g}\sqrt{\frac{(m-s)(3-s)}{2}}$$

image194:

$$D = G\exp\left[\frac{-Q_1^{\ 2} R_g^{\ 2}}{3-s}\right] Q_1^{\ (m-s)} = \frac{G}{R_g^{\ (m-s)}}\exp\left[-\frac{(m-s)}{2}\right]\left(\frac{(m-s)(3-s)}{2}\right)^{\frac{(m-s)}{2}}$$

image195:

$$I(q) = C / q^4 + background = 2\pi\Delta\rho S_V / q^4 + background$$

image197:

$$I(q) = (scale)\exp[-(q-q_0)^2/(2B^2)] + background$$

image198:

$$I(q) = \frac{(scale)}{(1+\left(\frac{q-q_0}{B}\right)^2)} + background$$

image200:

$$I(q) = scale\frac{2[(1+Ux)^{-1/U}+x-1]}{(1+U)x^2} + bkg$$

image202:

$$x = \frac{R_g^2 q^2}{1+2U}$$

image203:

$$P(Q) = 2\int_0^1 \mathrm{dx}\ (1-x)\exp\left[-\frac{Q^2a^2}{6}n^{2\nu}x^{2\nu}\right]$$

image204:

$$U = \frac{M_w}{M_n} - 1.$$

image204: (again)

$$P(Q) = 2\int_0^1 \mathrm{dx}\ (1-x)\exp\left[-\frac{Q^2a^2}{6}n^{2\nu}x^{2\nu}\right]$$

image206:

$$P(Q) = \frac{1}{\nu U^{1/2\nu}}\gamma(\frac{1}{2\nu},U)\ -\ \frac{1}{\nu U^{1/\nu}}\gamma(\frac{1}{\nu},U)$$

image207:

$$\gamma(x,U) = \int_0^U \mathrm{dt}\exp(-t)t^{x-1}$$

image208:

$$U = \frac{\mathrm{Q}^2\mathrm{a}^2\mathrm{n}^{2\nu}}{6} = \frac{\mathrm{Q}^2\mathrm{R}_g{}^2(2\nu+1)(2\nu+2)}{6}$$

image209:

$$\mathrm{R}_g{}^2 = \frac{\mathrm{a}^2\mathrm{n}^{2\nu}}{(2\nu+1)(2\nu+2)}$$

image210:

$$P(Q\to\infty) = \frac{1}{\nu\mathrm{U}^{1/2\nu}}\Gamma(\frac{1}{2\nu}) \;\; - \;\; \frac{1}{\nu\mathrm{U}^{1/\nu}}\Gamma(\frac{1}{\nu})$$

image211:

$$P(Q\to\infty) \sim \frac{1}{\nu\mathrm{U}^{1/2\nu}}\Gamma(\frac{1}{2\nu}) = \frac{\mathrm{m}}{\left(\mathrm{QR}_g\right)^m}\left[\frac{6}{(2\nu+1)(2\nu+2)}\right]^{m/2}\Gamma(m/2)$$

image212:

$$P(Q) = \frac{2}{Q^4R_g{}^4}\left[\exp(-Q^2R_g{}^2)-1+Q^2R_g{}^2\right]$$

image213:

$$\mathrm{I(Q)} = \frac{\mathrm{A}}{1+(\mathrm{Q}\xi_1)^{\mathrm{n}}} + \frac{\mathrm{C}}{1+(\mathrm{Q}\xi_2)^{\mathrm{m}}} + \mathrm{B}$$

image216:

$$I(q) = \begin{cases} \dfrac{A}{q^{m1}} & for\ q \le q_c \\ \dfrac{A\cdot q^{m1}/q^{m2}}{q^{m2}} & for\ q \le q_c \end{cases}$$

image218:

$$I(q) = Bkgd + \sum_{i=1}^{N} G_i\exp\left(-q^2R_{g,i}^2/3\right) + \frac{B_i\left[erf(qR_{g,i}/\sqrt{6})\right]^{3P_i}}{q^{P_i}}$$

image220:

$$I(q) = A + Bq$$

image222:

$$U(r) = \begin{cases} \infty, r < 2R \\ 0, r \geq 2R \end{cases}$$

image223:

$$U(r) = \begin{cases} \infty, r < 2R \\ -\varepsilon, 2R \leq r \leq 2R\lambda \\ 0, r \geq 2R \end{cases}$$

image225:

$$\tau = \frac{1}{12\varepsilon} \exp(u_o / kT)$$
$$\varepsilon = \Delta / (\sigma + \Delta)$$

image228:

$$U(r) = \begin{cases} \infty, r < \sigma \\ -U_0, \sigma \leq r \leq \sigma + \Delta \\ 0, r \geq \sigma + \Delta \end{cases}$$

image229:

$$I(Q) = I(0)_L \cdot \frac{1}{(1 + [((D+1)/3).Q^2 a_1^2])^{D/2}} + I(0)_G .\exp(-Q^2 a_2^2) + B$$

image233:

$$a_2^2 \approx \frac{R_g^2}{3}$$

image234:

$$I(Q) = N.V^2.(\Delta\rho)^2.P(Q) + B$$

image236:

$$P(Q)=\{[1+(Q^2.a)]^{D_m/2}\times[1+(Q^2.b)]^{(6-D_s-D_m)/2}\}^{-1}$$

$$a = R_g^2/(3.D_m/2)$$

$$b = r_g^2/[-3.(D_s-6+D_m)/2]$$

image237:

$$V_s = \pi(R+t)^2 \cdot (L+2t)$$

image239:

$$P(q)=\frac{scale}{V}[m_p^2(N+2\sum_{n=1}^{N-1}(N-n)\frac{\sin(qnl)}{qnl})(3\frac{\sin(qR)-qR\cos(qR)}{(qr)^3})^2]$$

linearpearl_eq1:

$$M_{0x} = M_0 cos\theta_M cos\phi_M$$

mox_eq:

$$M_{0y} = M_0 sin\theta_M$$

moy_eq:

$$M_{0z} = -M_0 cos\theta_M sin\phi_M$$

moz_eq:

$$I(q) = scale \times P(q)S(q) + bck$$
$$P(q) = F(qR)^2$$
$$F(x) = \frac{3[\sin(x) - x\cos(x)]}{x^3}$$
$$S(q) = \frac{\Gamma(D_m-1)\zeta^{(D_m-1)}}{\left[1+(q\zeta)^2\right]^{(D_m-1)/2}}\frac{\sin\left[(D_m-1)\tan^{-1}(q\zeta)\right]}{q}$$
$$scale = scale\_factor \times NV^2(\rho_{particle} - \rho_{solvent})^2$$
$$V = \frac{4}{3}\pi R^3$$

mass_fractal_eq1:

$$I(q) = scale \times P(q) + background$$

$$P(q) = \{[1+(q^2 a)]^{D_m/2} \times [1+(q^2 b)]^{(6-D_s-D_m)/2}\}^{-1}$$

$$a = R_g^2 / (3D_m / 2)$$

$$b = r_g^2 / [-3(D_s + D_m - 6)/2]$$

$$scale = scale\_factor \times NV^2 (\rho_{particle} - \rho_{solvent})^2$$

masssurface_fractal_eq1:

$$M_{0q_x} = (M_{0x} cos\phi - M_{0y} sin\phi) cos\phi$$

mqx:

$$M_{0q_y} = (M_{0y} sin\phi - M_{0x} cos\phi) sin\phi$$

mqy:

$$M_{\perp x'} = M_{0q_x} cos\theta_{up} + M_{0q_y} sin\theta_{up}$$

mxp:

$$M_{\perp y'} = M_{0q_y} cos\theta_{up} - M_{0q_x} sin\theta_{up}$$

myp:$$
M_{\perp y'} = M_{0q_y}\cos\theta_\text{up} - M_{0q_x}\sin\theta_\text{up}
$$

$$M_{\perp z'} = M_{0z}$$

mzp:$$
M_{\perp z'} = M_{0z}
$$

$$P(q) = \frac{P_0(q)}{V} = \frac{1}{V} F(q) F^*(q)$$

New Picture:$$
P(q) = \frac{P_o(q)}{V} = \frac{1}{V}F(q)F^*(q)
$$

$$f(x) = \frac{1}{Norm} \begin{cases} 1 & \text{for } |x - x_{mean}| \leq w \\ 0 & \text{for } |x - x_{mean}| > w \end{cases}$$

```
pd_image001:$$
f(x) = \frac{1}{\text{Norm}}\begin{cases}
1 & \text{for}\ |x-x_\text{mean}| \leq w \\
0 & \text{for}\ |x-x_\text{mean}| \gt w
\end{cases}
$$
```

$$\sigma = w/\sqrt{3}$$

```
pd_image002:$$
\sigma = w/\sqrt{3}
$$
```

$$PD = \sigma/x_{mean}$$

```
pd_image003:$$
PD = \sigma/x_\text{mean}
$$
```

$$f(x) = \frac{1}{Norm}\exp\left(-\frac{(x-x_{mean})^2}{2\sigma^2}\right)$$

```
pd_image005:$$
f(x) = \frac{1}{\text{Norm}}\exp\left(-\frac{x-x_\text{mean})^2}{2\sigma^2}\right)
$$
```

$$f(x) = \frac{1}{Norm}\frac{1}{xp}\exp\left(-\frac{(\ln(x)-\mu)^2}{2p^2}\right)$$

```
pd_image007:$$
f(x) = \frac{1}{\text{Norm}}\frac{1}{xp}\exp\left(-\frac{(\ln(x)-\mu)^2}{2p^2}\right)
$$
```

$$PD = p$$

```
pd_image008:$$
PD = p
$$
```

$$p = \sigma/x_{med}$$

```
pd_image009:$$
p = \sigma/x_\text{med}
$$
```

$$f(x) = \frac{1}{Norm}(z+1)^{z+1}(x/x_{mean})^z\frac{\exp[-(z+1)x/x_{mean}]}{x_{mean}\Gamma(z+1)}$$

```
pd_image011:$$
f(x) = \frac{1}{\text{Norm}}(z+1)^{z+1}(x/x_\text{mean})^z
        \frac{\exp\left[-(z+1)x/x_\text{mean}\right]}{x_\text{mean}\Gamma(z+1)}
$$
```

$$p = \sigma / x_{mean}$$

```
pd_image012:$$
p=\sigma/x_\text{mean}
$$
```

$$P(q) = \frac{scale}{V}\cdot\frac{(S_{zz}(q)+S_{rr}(q)+S_{rz}(q))}{(M\cdot m_r + N\cdot m_z)^2} + bkg$$

```
pearl_eq1:$$
P(q) = \frac{\text{scale}}{V} \cdot \frac{S_{zz}(q) + S_{rr}(q) + S_{rz}(q)}
  {(M\cdot m_r + N\cdot m_z)^2} + \text{background}
$$
```

$$S_{zz}(q) = 2m_z^2\psi^2(q)\left[\frac{N}{1-\sin(qA)/qA} - \frac{N}{2} - \frac{1-(\sin(qA)/qA)^N}{(1-\sin(qA)/qA)^2}\cdot\frac{\sin(qA)}{qA}\right]$$

```
pearl_eq2:$$
S_{zz}(q) = 2m_z^2\psi^2(q)\left[\frac{N}{1-\sin(qA)/qA} - \frac{N}{2} -
        \frac{1-(\sin(qA)/qA)^N}{(1-\sin(qA)/qA)^2}\cdot\frac{\sin(qA)}{qA}\right]
$$
```

$$S_{rr}(q) = m_r^2\left[M\left\{2\Lambda(q) - \left(\frac{\sin(ql/2)}{ql/2}\right)\right\} + \frac{2M\beta^2(q)}{1-\sin(qA)/qA} - 2\beta^2(q)\frac{1-(\sin(qA)/qA)^M}{(1-\sin(qA)/qA)^2}\right]$$

```
pearl_eq3:$$
S_{rr}(q) = m_r^2\left[M\left\{2\Lambda(q) -\frac{\sin(ql/2)}{ql/2}\right\} +
        \frac{2M\beta^2(q)}{1-\sin(qA)/qA} -
  2\beta^2(q)\frac{1-(\sin(qA)/qA)^M}{(q-sin(qA)/qA)^2}\right]
$$
```

$$S_{rz}(q) = m_r\beta(q)\cdot m_z\psi(q)\cdot 4\left[\frac{N-1}{1-\sin(qA)/qA} - \frac{1-(\sin(qA)/qA)^{N-1}}{(1-\sin(qA)/qA)^2}\cdot\frac{\sin(qA)}{qA}\right]$$

```
pearl_eq4:$$
S_{rz}(q) = m_r\beta(q)\cdot m_z\psi(q)\cdot
  4\left[\frac{N-1}{1-\sin(qA)/qA} - \frac{1-(\sin(qA)/qA)^{N-1}}{(q-sin(qA)/qA)^2}
  \cdot\frac{\sin(qA)}{qA}\right]
$$
```

$$\psi(q) = 3 \cdot \frac{\sin(qR) - (qR)\cdot\cos(qR)}{(qR)^3}$$

pearl_eq5:$$
\psi(q) = 3\cdot\frac{\sin(qR) - (qR)\cdot\cos(qR)}{(qR)^3}
$$

$$\Lambda(q) = \frac{\int_0^{ql} \frac{\sin(t)}{t} dt}{ql}$$

pearl_eq6:$$
\Lambda(q) = \frac{1}{ql} \int_0^{ql}\frac{\sin(t)}{t} dt
$$

$$\beta(q) = \frac{\int_{qR}^{q(A-R)} \frac{\sin(t)}{t} dt}{ql}$$

pearl_eq7:$$
\beta(q) = \frac{1}{ql} \int_{qR}^{q(A{-}R)}\frac{\sin(t)}{t} dt
$$

$$I(q) = (\Delta\rho)^2 V \int_0^{\pi/2} d\psi \sin\psi \ \text{sinc}^2\left(\frac{qd\cos\psi}{2}\right)\left[(\mathcal{S}_0^2 + \mathcal{C}_0^2) + 2\sum_{n=1}^{\infty}(\mathcal{S}_n^2 + \mathcal{C}_n^2)\right]$$

pringle_eqn_1:$$
I(q) = (\Delta\rho)^2V\int_0^{\pi/2} d\psi \sin \psi
\text{sinc}^2\left(\frac{qd\cos\psi)}{2}\right)
\left[(S_o^2 + C_o^2 + 2 \sum_{n=1}^\infty (S_n^2 + C_n^2)\right]
$$

$$\mathcal{C}_n = \int_0^R r dr \cos(qr^2\alpha\cos\psi) \mathrm{J}_n(qr^2\beta\cos\psi)\mathrm{J}_{2n}(qr\sin\psi)$$

$$\mathcal{S}_n = \int_0^R r dr \sin(qr^2\alpha\cos\psi) \mathrm{J}_n(qr^2\beta\cos\psi)\mathrm{J}_{2n}(qr\sin\psi)$$

```
pringle_eqn_2: $$
\begin{eqnarray}
C_n &=& \int_0^R r dr \cos(qr^2 \alpha \cos \psi)
   \text{J}_n(qr^2\beta\cos\psi)\text{J}_{2n}(qr\sin\psi) \\
S_n &=& \int_0^R r dr \sin(qr^2\alpha \cos \phi)
   \text{J}_n(qr^2\beta\cos\psi)\text{J}_{2n}(qr\sin\psi)
    \end{eqnarray}
$$
```

$$P(q) = \frac{1}{V^2}\frac{2}{\pi} \times \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} A_{P\Delta}^2(q) \sin\theta \, d\theta \, d\varphi$$

```
RectangularHollowPrism_1: $$
P(q) = \frac{1}{V^2}\frac2\pi \times
   \int\limits_0^\tfrac\pi2\int\limits_0^\tfrac\pi2 A_{P\Delta}^2(q)
   \sin\theta d\theta d\phi
$$
```

$$\begin{aligned} A_{P\Delta}(q) = A\ B\ C \times & \frac{\sin\left(q\frac{C}{2}\cos\theta\right)}{q\frac{C}{2}\cos\theta}\frac{\sin\left(q\frac{A}{2}\sin\theta\sin\varphi\right)}{q\frac{A}{2}\sin\theta\sin\varphi}\frac{\sin\left(q\frac{B}{2}\sin\theta\cos\varphi\right)}{q\frac{B}{2}\sin\theta\cos\varphi} - 8\left(\frac{A}{2}-\Delta\right)\left(\frac{B}{2}-\Delta\right)\left(\frac{C}{2}-\Delta\right) \\ \times & \frac{\sin\left[q\left(\frac{C}{2}-\Delta\right)\cos\theta\right]}{q\left(\frac{C}{2}-\Delta\right)\cos\theta}\frac{\sin\left[q\left(\frac{A}{2}-\Delta\right)\sin\theta\sin\varphi\right]}{q\left(\frac{A}{2}-\Delta\right)\sin\theta\sin\varphi}\frac{\sin\left[q\left(\frac{B}{2}-\Delta\right)\sin\theta\cos\varphi\right]}{q\left(\frac{B}{2}-\Delta\right)\sin\theta\cos\varphi} \end{aligned}$$

```
RectangularHollowPrism_2:$$
\begin{eqnarray}
A_{P\Delta}(q) = A B C &\times&
    \frac{\sin\left(\tfrac12qC\cos\theta\right)}{\tfrac12qC\cos\theta}
      \frac{\sin\left(\tfrac12qA\sin\theta\sin\phi)\right)}{\tfrac12qA\sin\theta\sin\phi}
      \frac{\sin\left(\tfrac12qB\sin\theta\cos\phi\right)}{\tfrac12qB\sin\theta\cos\phi}
    - (A{-}2\Delta)(B{-}2\Delta)(C{-}2\Delta) \\
    &\times&
  \frac{\sin\left(\tfrac12q(C{-}2\Delta)\cos\theta\right)}
    {\tfrac12q(C{-}2\Delta)\cos\theta}
  \frac{\sin\left(\tfrac12q(A{-}2\Delta)\sin\theta\sin\phi)\right)}
    {\tfrac12q(A{-}2\Delta)\sin\theta\sin\phi}
  \frac{\sin\left(\tfrac12q(B{-}2\Delta)\sin\theta\cos\phi\right)}
    {\tfrac12q(B{-}2\Delta)\sin\theta\cos\phi}
\end{eqnarray}
$$
```

$$V = ABC - (A - 2\Delta)(B - 2\Delta)(C - 2\Delta)$$

RectangularHollowPrism_3:$$
V = ABC - (A-2\Delta)(B-2\Delta)(C-2\Delta)
$$

$$I(q) = \text{scale} \times V \times (\rho_{\text{pipe}} - \rho_{\text{solvent}})^2 \times P(q)$$

RectangularHollowPrism_4:$$
I(q) = \text{scale} \times V \times (\rho_\text{pipe} - \rho_\text{solvent})^2 \times P(q)
$$

$$P(q) = \frac{1}{V^2}\frac{2}{\pi} \times \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} [A_L(q) + A_T(q)]^2 \sin\theta \, d\theta \, d\varphi$$

RectangularHollowPrismInfThinWalls_1:$$
P(q) = \frac{1}{V^2}\frac2\pi \times
  \int\limits_0^\tfrac\pi2\int\limits_0^\tfrac\pi2 [A_L(q) + A_T(q)]^2
   \sin\theta\,d\theta d\phi
$$

$$V = 2AB + 2AC + 2BC$$

RectangularHollowPrismInfThinWalls_2:$$
V = 2 A B + 2 A C + 2 B C
$$

$$A_L(q) = 8 \times \frac{\sin\left(q\frac{A}{2}\sin\varphi\sin\theta\right)\sin\left(q\frac{B}{2}\cos\varphi\sin\theta\right)\cos\left(q\frac{C}{2}\cos\theta\right)}{q^2\sin^2\theta\sin\varphi\cos\varphi}$$

RectangularHollowPrismInfThinWalls_3:$$
A_L(q) = 8 \times \frac{\sin\left(\tfrac12qA\sin\phi\sin\theta\right)
   \sin\left(\tfrac12qB\cos\phi\sin\theta\right)
   \cos\left(\tfrac12qC\cos\theta\right)}{q^2\sin^2\theta\sin\phi\cos\phi}
$$

$$A_T(q) = A_F(q) \times \frac{2\sin\left(q\frac{C}{2}\cos\theta\right)}{q\cos\theta}$$

RectangularHollowPrismInfThinWalls_4:$$
A_T(q) = A_F(q) \times \frac{2 \sin\left(\tfrac12qC\cos\theta\right)}{q\cos\theta}
$$

$$A_F(q) = 4\frac{\cos\left(q\frac{A}{2}\sin\varphi\sin\theta\right)\sin\left(q\frac{B}{2}\cos\varphi\sin\theta\right)}{q\ \cos\varphi\ \sin\theta} + 4\frac{\sin\left(q\frac{A}{2}\sin\varphi\sin\theta\right)\cos\left(q\frac{B}{2}\cos\varphi\sin\theta\right)}{q\ \sin\varphi\ \sin\theta}$$

RectangularHollowPrismInfThinWalls_5:$$
A_F(q) = 4 \frac{\cos\left(\tfrac12qA\sin\phi\sin\theta\right)
\sin\left(\tfrac12qB\cos\phi\sin\theta\right)}{q\cos\phi\sin\theta} +
4 \frac{\sin\left(\tfrac12qA\sin\phi\sin\theta\right)
\cos\left(\tfrac12qB\cos\phi\sin\theta\right)}{q\sin\phi\sin\theta}
$$

$$I(q) = \text{scale} \times V \times (\rho_{\text{pipe}} - \rho_{\text{solvent}})^2 \times P(q)$$

RectangularHollowPrismInfThinWalls_6:$$
I(q) = \text{scale} \times V \times (\rho_\text{pipe} - \rho_\text{solvent})^2 \times P(q)
$$

$$A_P(q) = \frac{\sin\left(q\frac{C}{2}\cos\theta\right)}{q\frac{C}{2}\cos\theta} \times \frac{\sin\left(q\frac{A}{2}\sin\theta\sin\varphi\right)}{q\frac{A}{2}\sin\theta\sin\varphi} \times \frac{\sin\left(q\frac{B}{2}\sin\theta\cos\varphi\right)}{q\frac{B}{2}\sin\theta\cos\varphi}$$

RectangularPrism_1:$$
A_P(q) = \frac{\sin\left(\tfrac12qC\cos\theta\right)}{\tfrac12qC\cos\theta} \times
\frac{\sin\left(\tfrac12qA\sin\theta\sin\phi)\right)}{\tfrac12qA\sin\theta\sin\phi} \times
\frac{\sin\left(\tfrac12qB\sin\theta\cos\phi\right)}{\tfrac12qB\sin\theta\cos\phi}
$$

$$P(q) = \frac{2}{\pi} \times \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} A_P^2(q)\ \sin\theta\ d\theta\ d\varphi$$

RectangularPrism_2:$$
P(q) = \frac2\pi \times
\int\limits_0^\tfrac\pi2\int\limits_0^\tfrac\pi2 A^2_P(q) \sin\theta\, d\theta d\phi
$$

$$I(q) = \text{scale} \times V \times (\rho_{\text{pipe}} - \rho_{\text{solvent}})^2 \times P(q)$$

RectangularPrism_3:$$
I(q) = \text{scale} \times V \times (\rho_\text{pipe} - \rho_\text{solvent})^2 \times P(q)
$$

$$I_0(q) = scale \cdot (\rho_{poly} - \rho_{solv})^2 \left[\frac{6\pi\phi_{core}}{Q^2} \frac{\Gamma^2}{\delta^2_{poly} R_{core}} \exp(-Q^2\sigma^2) \right] + background$$

secondmeq1:$$
I_o(q) = \text{scale} \cdot(\rho_\text{poly}-\rho_\text{solvent})^2
\left[\frac{6\pi\phi_\text{core}}{Q^2}\frac{\Gamma^2}{\delta_\text{poly}^2R_\text{core}}
\exp(-Q^2\sigma^2)\right] + \text{background}
$$

$$\beta_{\pm\pm} = \beta_N \mp D_M M_{\perp x'}$$

sld1:$$
\beta_{\pm\pm} = \beta_N \mp D_M M_{\perp x'}
$$

$$\beta_{\pm\mp} = -D_M \left(M_{\perp y'} \pm i M_{\perp z'} \right)$$

sld2:$$
\beta_{\pm\mp} = -D_M(M_{\perp y'} \pm i M_{\perp z'})
$$

$$I_s = \frac{1}{Norm} \int_{-\infty}^{\infty} dv W_v(v) \int_{-\infty}^{\infty} du W_u(u) I(\sqrt{(q+v)^2 + |u|^2})$$

sm_image002:$$
I_s = \frac{1}{\text{Norm}} \int_{-\infty}^\infty dv\,W_v(v)
\int_{-\infty}^\infty du W_u(u) I\left(\sqrt{(q+v)^2 + |u|^2}\right)
$$

$$\int_{-\infty}^{\infty} dv W_v(v) \int_{-\infty}^{\infty} du W_u(u)$$

sm_image003:$$
\int_{-\infty}^\infty dv\,W_v(v) \int_{-\infty}^\infty du W_u(u)
$$

$W_v(v)$

sm_image004:$$
W_v(v)
$$

$W_u(u)$

sm_image005:$$
W_u(u)
$$

$W_v(v) = \delta(|v| \leq \Delta q_v)$

sm_image006:$$
W_v(v) = \delta(|v| \leq \Delta q_v)
$$

$W_u(u) = \delta(|u| \leq \Delta q_u)$

sm_image007:$$
W_u(u) = \delta(|u| \leq \Delta q_u)
$$

$\Delta q_\alpha = \int_0^\infty d\alpha W_\alpha(\alpha)$

sm_image008 and sm_image009:$$
\Delta q_\alpha = \int_0^\infty d\alpha W_\alpha(\alpha)
$$

$\alpha = v$

sm_image010:$$
\alpha = v
$$

Δq_u

sm_image011:$$
\Delta q_u
$$

Δq_v

sm_image012:$$
\Delta q_v
$$

$$I_s(q) = \frac{2}{Norm} \int_{-\Delta q_v}^{\Delta q_v} dv \int_0^{\Delta q_u} du \, \mathrm{I}(\sqrt{(q+v)^2 + u^2}\,)$$

sm_image013:$$
I_s(q) = \frac{2}{\text{Norm}} \int_{-\Delta q_v}^{\Delta q_v} dv \int_0^{\Delta q_u} du
I\left(\sqrt{(q_v)^2 + u^2}\right)
$$

Δq_v

sm_image014:$$
\Delta q_v
$$

Δq_u

sm_image015:$$
\Delta q_u
$$

$$I_s(q) \approx \int_0^{\Delta q_u} du \, \mathrm{I}(\sqrt{q^2 + u^2}\,) = \int_0^{\Delta q_u} d(\sqrt{q'^2 - q^2}\,)\, \mathrm{I}(q')$$

sm_image016:$$
I_s(q) \approx \int_0^{\Delta q_u} du I\left(\sqrt{q^2+u^2}\right)
= \int_0^{\Delta q_u} d\left(\sqrt{q'^{2} – q^2}\right) I(q')
$$

$$
I_s(q_i) \approx \sum_{j=i}^{N-1}\left[\sqrt{q_{j+1}^2 - q_i^2} - \sqrt{q_j^2 - q_i^2}\right] I(q_j) \approx \sum_{j=i}^{N-1} W_{ij} I(q_j)
$$

```
sm_image017:$$
I_s(q) \approx \sum_{j=i}^{N-1}\left[\sqrt{q_{j+1}^2 – q_i^2} - \sqrt{q_j^2 – q_i^2}\right] I(q_j)
        \approx \sum_{j=i}^{N-1}W_{ij}I(q_j)
$$
```

```
sm_image018:$$
W_{ij}
$$
```

$$
I_s(q_i) \approx \sum_{j=p}^{N-1} [q_{j+1} - q_i] \, I(q_j) \approx \sum_{j=p}^{N-1} W_{ij} I(q_j)
$$

```
sm_image019:$$
I_s(q_i) \approx \sum_{j=p}^{N-1}[q_{j+1} - q_i] I(q_j) \approx\sum_{j=p}^{N-1} W_{ij} I(q_j)
$$
```

$$
\begin{aligned}
I_s(q_i) &\approx \sum_{j=p}^{N-1} \sum_{k=-L}^{L} \left[\sqrt{q_{j+1}^2 - (q_i + (k\Delta q_v/L))^2} - \sqrt{q_j^2 - (q_i + (k\Delta q_v/L))^2}\right] (\Delta q_v/L) I(q_j) \\
&\approx \sum_{j=p}^{N-1} W_{ij} I(q_j)
\end{aligned}
$$

```
sm_image020:$$
\begin{eqnarray}
I_s(q_i) &\approx& \sum_{j=p}^{N-1}\sum_{k=-L}^L
  \left[\sqrt{q_{j+1}^2 - (q_i+k\Delta q_\nu/L)^2}
  - \sqrt{q_j^2 - (q_i + k\Delta q_\nu/L)^2}\right](\Delta q_\nu/L)I(q_j) \\
&\approx& \sum_{j=p}^{N-1}W_{ij}I(q_j)
\end{eqnarray}
$$
```

$$
\begin{aligned}
I_s(q_i) &\approx \sum_{j=0}^{N-1}[\operatorname{erf}(q_{j+1})-\operatorname{erf}(q_j)]I(q_j)\\
&\approx \sum_{j=0}^{N-1} W_{ij} I(q_j)
\end{aligned}
$$

sm_image021:$$
\DeclareMathOperator\erf{erf}
\begin{eqnarray}
I_s(q_i)&\approx&\sum_{j=0}^{N-1} [\erf(q_{j+1})- \erf(q_j)]I(q_j) \\
&\approx&\sum_{j=0}^{N-1}W_{ij}I(q_j)
\end{eqnarray}
$$

$$
\begin{aligned}
I_s(x_0,y_0) &= A\iint dx'dy'\exp\left[-\left(\frac{(x'-x_0{}')^2}{2\sigma_{x_0{}'}^2}+\frac{(y'-y_0{}')^2}{2\sigma_{y_0{}'}^2}\right)\right]I(x',y')\\
&= A\sigma_{x_0{}'}\sigma_{y_0{}'}\iint dXdY\exp\left[-\frac{(X^2+Y^2)}{2}\right]I(\sigma_{x_0{}'}X+x_0{}',\sigma_{y_0{}'}Y+y_0{}')\\
&= A\sigma_{x_0{}'}\sigma_{y_0{}'}\iint dRd\Theta\,\mathrm{R}\exp\left(-\frac{R^2}{2}\right)I(\sigma_{x_0{}'}Rcos\Theta+x_0{}',\sigma_{y_0{}'}Rsin\Theta+y_0{}')
\end{aligned}
$$

sm_image022:$$
\begin{eqnarray}
I_s(x_o,y_o) &=& A \iint dx’dy’\exp\left[-\left(\frac{x’-x_o’)^2}{2\sigma_{x_o’}^2}
+\frac{y’-y_o’)^2}{2\sigma_{y_o’}^2}\right)\right]I(x’,y’) \\
&=& A\sigma_{x_o’}\sigma_{y_o’}\iint dXdY\exp\left[-\frac{X^2+Y*2}{2}\right]
I(\sigma_{x_o’}X + x_o’, \sigma_{y_o’}Y + y_o’) \\
&=& A\sigma_{x_o’}\sigma_{y_o’}\iint dR d\Theta R\exp\left(-\frac{R^2}{2}\right)
I(\sigma_{x_o’} R\cos\Theta + x_o’, \sigma_{y_o’} R\sin\Theta + y_o’)
\end{eqnarray}
$$

$$
\begin{aligned}
I_s(x_0,y_0) &\approx A\sigma_{x_0{}'}\sigma_{y_0{}'}\sum_{i}^{nbins}\Delta\Theta\left[\exp\left(\frac{(R_i-\Delta R/2)^2}{2}\right)-\exp\left(\frac{(R_i+\Delta R/2)^2}{2}\right)\right]I(\sigma_{x_0{}'}R_i\cos\Theta_i+\\
&\quad x_0{}',\sigma_{y_0{}'}R_i\sin\Theta_i+y_0{}')\\
&\approx \sum_{i}^{nbins} W_i\, I(\sigma_{x_0{}'}R_i\cos\Theta_i+x_0{}',\sigma_{y_0{}'}R_i\sin\Theta_i+y_0{}')
\end{aligned}
$$

```
sm_image024:$$
\begin{eqnarray}
I_s(x_o,y_o) & \approx & A\sigma_{x_o'}\sigma_{y_o'} \sum_i^{\text{nbins}} \Delta\Theta
 \left[\exp\left(\frac{(R_i-\Delta R/2)^2}{2}\right) - \exp\left(\frac{(R_i + \Delta
R/2)^2}{2}\right)
 \right] I(\sigma_{x_o'} R_i \cos \Theta_i + x_o',  \sigma_{y_o'} R_i \sin \Theta_i + y_o') \\
&\approx& \sum_i^{\text{nbins}} W_i I(\sigma_{x_o'} R_i \cos \Theta_i + x_o',
     \sigma_{y_o'} R_i \sin \Theta_i + y_o')
\end{eqnarray}
$$
```

$$I_s(x_0, y_0) \approx \sum_i^{nbins} W_i I(x' \cos\theta - y' \sin\theta, x' \sin\theta + y' \cos\theta)$$

```
sm_image025:$$
I_s(x_o, y_0) \approx \sum_i^{\text{nbins}} W_i I(x'\cos\theta - y'\sin\theta,
   x'\sin\theta + y'\cos\theta)
$$
```

$$\begin{aligned}
x' &= \sigma_{x_0'} R_i \cos\Theta_i + x_0' \\
y' &= \sigma_{y_0'} R_i \sin\Theta_i + y_0' \\
x_0' &= q = \sqrt{x_0^2 + y_0^2} \\
y_0' &= 0
\end{aligned}$$

```
sm_image026:$$
\begin{eqnarray}
x' &=& \sigma_{x_o'}R_i\cos\Theta_i + x_o' \\
y' &=& \sigma_{y_o'}R_i\sin\Theta_i + y_o' \\
x_o' &=& q = \sqrt{x_o^2 + y_o^2} \\
y_o' &=& 0
\end{eqnarray}
$$
```

$$I_s(x_0, y_0) \approx \sum_i^{nbins} W_i I(x', y')$$

sm_image027:$$
I_s(x_o,y_o) \approx \sum_i^\text{nbins} W_i I(x’, y’)
$$

$$
\begin{aligned}
x' &= \sigma_{x'_0} R_i \cos\Theta_i + x'_0 \\
y' &= \sigma_{y'_0} R_i \sin\Theta_i + y'_0 \\
x'_0 &= x_0 = q_x \\
y'_0 &= y_0 = q_y
\end{aligned}
$$

sm_image028:$$
\begin{eqnarray}
x’ &=& \sigma_{x_o’}R_i\cos\Theta_i + x_o’ \\
y’ &=& \sigma_{y_o’}R_i\sin\Theta_i + y_o’ \\
x_o’ &=& x_o = q_x\\
y_o’ &=& y_o = q_y
\end{eqnarray}
$$

$$I(q) = \frac{2}{fv^2}[v - 1 + \exp(-v) + \frac{f-1}{2}[1 - \exp(-v)]^2]$$

star1:$$
I(q) = \frac{2}{fv^2}\left[v-1 + \exp(-v) \frac{f-1}{2}[1-\exp(-v)]^2\right]
$$

$$v = \frac{u^2 f}{(3f-2)}$$

star2:$$
v = \frac{u^2f}{3f-2}
$$

$$u = < R_g^2 > q^2$$

star3:$$
u = \langle R_g^2 \rangle q^2
$$

$$I(q) = scale \times P(q)S(q) + background$$

$$P(q) = F(qR)^2$$

$$F(x) = \frac{3[\sin(x) - x\cos(x)]}{x^3}$$

$$S(q) = \frac{\Gamma(5-D_s)\zeta^{(5-D_s)}}{\left[1+(q\zeta)^2\right]^{(5-D_s)/2}}\frac{\sin\left[(D_s-5)\tan^{-1}(q\zeta)\right]}{q}$$

$$scale = scale\_factor \times NV^2(\rho_{particle} - \rho_{solvent})^2$$

$$V = \frac{4}{3}\pi R^3$$

surface_fractal_eq1:$$
\begin{eqnarray}
I(q) &=& \text{scale} \times P(q) S(q) + \text{background} \\
P(q) &=& F(qR)^2 \\
F(x) &=& \frac{3\,[\sin(x) - x\cos(x)]}{x^3} \\
S(q) &=& \frac{\Gamma(5-D_s)\zeta^{(5-D_s)}}{\left[1+(q\zeta)^2\right]^{(5-D_s)/2}}
\frac{\sin\left[(D_s-5)\tan^{-1}(q\zeta)\right]}{q} \\
\text{scale} &=& \text{scale factor} \times NV^2 (\rho_\text{particle} -
\rho_\text{solvent})^2 \\
V &=& \tfrac43\pi R^3
\end{eqnarray}
$$